QB365 QUESTION BANK 365

# 12th Standard- Business Maths

**TENTATIVE ANSWER KEY**

PART - I

1. d 0.2
2. b Consistent and it has Infinitely many solutions
3. b $(-a, 0)$
4. b $\frac{\sqrt{17}}{4}$
5. b 3
6. c $y = 3x$
7. c $x^y \log x$
8. b Concave downward
9. d $\frac{\pi}{4}$
10. c $\log|x+1| + k$
11. c $e^{-\frac{1}{2}x}\left[A\cos\frac{\sqrt{3}}{2}x + B\sin\frac{\sqrt{3}}{2}x\right]$
12. a 2 and 2
13. a 59
14. a $f(x+h) - f(x)$
15. d
16. c

11

18. a 45
19. a Erratic Variation
20. d None of these

PART - II

21. Order of A is $3 \times 3$

$\therefore \rho(A) \le 3$

Consider only third order minor is zero.

$\therefore \rho(A) < 2$

Consider the second order minors. obviously they are all zero. $\therefore \rho(A) \le 1$

Since A is a non-zero matrix

$\therefore \rho(A) = 1$

22. $\frac{x^2}{16} - \frac{y^2}{9} = 1$

$a^2 = 16$, $a = 4$ $\quad$ $b^2 = 9$, $b = 3$

$e = \frac{5}{4}$

Length of transverse axis $= 2a = 8$

length of Conjugate axis $= 2b = 6$.

23. Area $A = \int_1^2 (3x^2 - 4x + 5)\,dx$

$$= \left[\frac{3x^3}{3} - \frac{4x^2}{2} + 5x\right]_1^2$$

$$= 6 \text{ sq. units.}$$

24. $u = x^3 + y^3 + z^3 - 3xyz$

$$\frac{\partial u}{\partial x} = 3x^2 - 3yz$$

$$x\frac{\partial u}{\partial x} = 3x^3 - 3xyz$$

$$\frac{\partial u}{\partial y} = 3y^2 - 3xz$$

$$y\frac{\partial u}{\partial y} = 3y^3 - 3xyz$$

$$\frac{\partial u}{\partial z} = 3z^2 - 3xy$$

$$z\frac{\partial u}{\partial z} = 3z^3 - 3yxz$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$$

$$= 3(x^3 + y^3 + z^3 - 3xyz)$$

$$= 3u.$$

25. $y = a\cos(mx+b)$

$$\frac{dy}{dx} = -ma\sin(mx+b)$$

$$\frac{d^2y}{dx^2} = -m^2a\cos(mx+b) = -m^2y$$

$$\therefore \frac{d^2y}{dx^2} + m^2y = 0$$

26. Let $y = ax+b$ be the line of best fit.

Normal Equations are

$$b\Sigma x = \Sigma xy$$

$$a\Sigma x + nb = \Sigma y$$

$$30a + 10b = 90 \quad \text{— (1)}$$

$$10a + 5b = 25 \quad \text{— (2)}$$

Solve (1) and (2)

$a = 4 \quad b = -3$

$\therefore y = 4x - 3$

$\therefore$ slope $= 4$, $y$-intercept $= -3$

27. Given $n = 5$

$$P(X=3) = P(X=4)$$

$$5C_3\, p^3q^2 = 5C_4\, p^4q$$

$$2q = p$$

$$3q = 1$$

$$\boxed{q = 1/3} \quad \boxed{p = 2/3}$$

28. $\bar{x} = 0.824$, $S = 0.042$

95% Confidence limits are

$$\bar{x} \pm (Z_c)\frac{S}{\sqrt{n}} = 0.824 \pm 1.96\left(\frac{0.042}{\sqrt{200}}\right)$$

$$= 0.824 \pm 0.00582$$

95% Confidence interval for estimating $\mu$ is $(0.818,\ 0.829)$.

29.

| year | production | Semi Total | Semi Average |
|---|---|---|---|
| 1987 | 90 | | |
| 1988 | 110 | 330 | 110 |
| 1989 | 130 | | |
| 1990 | 150 | | |
| 1991 | 100 | | |
| 1992 | 150 | 450 | 150 |
| 1993 | 200 | | |

Difference between middle periods $\left.\right\} = 1992 - 1988 = 4$

Difference between Semi averages $\left.\right\} = 150 - 110 = 40$

Annual increase in trend $= \frac{40}{4} = 10$

| Year | 1987 | 1988 | 1989 | 1990 | 1991 | 1992 | 1993 |
|---|---|---|---|---|---|---|---|
| Trend | 100 | 110 | 120 | 130 | 140 | 150 | 160 |

30 $y^2 = 20x$ — (1)

$y' = \frac{10}{y}$

$m = \tan\theta$

$m = \tan 60°$

$m = \sqrt{3}$

At $(x_1, y_1)$,

$m = \frac{10}{y_1}$

$y_1 = \frac{10}{\sqrt{3}}$

(1) $\Rightarrow x_1 = \frac{y_1^2}{20} \Rightarrow x_1 = 5/3$

Equation of tangent is

$y - y_1 = m(x - x_1)$

$y - \frac{10}{\sqrt{3}} = \sqrt{3}\,(x - 5/3)$

$3x - \sqrt{3}y + 5 = 0$

PART — III

31. $A = \begin{pmatrix} 1 & 2 & k \\ 3 & 4 & 6 \\ 1 & 1 & 1 \end{pmatrix}$

The homogeneous equation to have only trivial solution

$\begin{vmatrix} 1 & 2 & k \\ 3 & 4 & 6 \\ 1 & 1 & 1 \end{vmatrix} \neq 0 \Rightarrow k \neq 4$

32. $y = \frac{1}{10}x^2 - 3x + 50$

$(x - 15)^2 = 10(y - 27.5)$

$X^2 = 10Y$

$X = x - 15 \quad Y = y - 27.5$

$a = 2.5$

The average variable cost curve as a parabola whose vertex is $x = 15,\ y = 27.5$

The output and average cost at the vertex are 15 kgs and Rs. 27.50 resp.

33. $f(x) = \frac{500}{x},\ x_0 = 20 \quad \Delta x = 0.5$

$\frac{\Delta y}{\Delta x} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$

$\frac{f(20.5) - 20}{0.5} = -1.22$

The negative sign indicates that y decreases per unit increases in x.

$y = \frac{500}{x}$

$\frac{dy}{dx} = \frac{-500}{x^2}$

At $x = 20,\ \frac{dy}{dx} = -1.25$

The negative sign indicates the decrease rate of change with respect to x.

34. $f(x) = x^3 - 27x + 108$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

$f'(x) = 0 \Rightarrow x = \pm 3$

when $x = 3$, $f''(x) > 0$

$f(x)$ is minimum.

Minimum value $= 54$

when $x = -3$, $f''(x) < 0$

$f(x)$ is maximum.

Maximum value $= 162$

Maximum value of $f(x)$ is greater than its minimum value.

35. $MC = 4 + 0.08x$

$C(x) = \int (MC)\,dx + k_1$

$= 4x + 0.04x^2 + k_1$

when $x = 0$, $C = 0$

$\Rightarrow k_1 = 0$

$C(x) = 4x + 0.04x^2$

Given $MR = 12$

$R(x) = \int MR\,dx + k_2 = 12x + k_2$

Revenue $= 0$ when $x = 0$

$k_2 = 0$

$\Rightarrow R(x) = 12x$

Total profit function

$P(x) = R(x) - C(x)$

$= 8x - 0.04x^2$

36. Given $x_0 = 6$, $x_1 = 7$, $x_2 = 10$, $x_3 = 12$ and $x = 11$

$y_0 = 13$, $y_1 = 14$, $y_2 = 15$, $y_3 = 17$

Using Lagrange's formula,

$$y = 13\frac{4(1)(-1)}{(-1)(-4)(-6)} + 14\frac{(5)(1)(-1)}{(1)(-3)(-5)} + 15\frac{(5)(4)(-1)}{(4)(3)(-2)} + 17\frac{(5)(4)(1)}{(6)(5)(2)}$$

$y = 15.6666$

37. i) $E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \frac{1}{2}\int_{-1}^{1} x\,dx$

$= 0$

ii) $E(X^2) = \int_{-1}^{1} x^2 f(x)\,dx$

$= \int_{-1}^{1} \frac{x^2}{2}\,dx = \frac{1}{3}$

iii) $Var(X) = E(X^2) - [E(X)]^2$

$= \frac{1}{3}$

38. Sample size $n = 1000$

$p = \frac{x}{n} = \frac{320}{1000} = 0.32$

$q = 0.68$

$S.E(p) = \sqrt{\frac{pq}{n}} = 0.0147$

95% Confidence limits for population proportion P

$p \pm (1.96)\,S.E(p) = 0.32 \pm 0.028$

$\Rightarrow 0.292$ and $0.348$

$\therefore$ TV viewers of this programme lie between 29.2% and 34.8%.

39. $\Sigma P_0 q_0 = 160$, $\Sigma P_1 q_0 = 200$

Laspeyre's Index $= P_{01}^{L} = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100$

$= 125$

40. put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = v^2 - v$$

$$\frac{dv}{(v-1)^2 - 1^2} = \frac{dx}{x}$$

Integrating, we have

$$\frac{1}{2}\log\left[\frac{(v-1)-1}{(v+1)+1}\right] = \log x + \log c$$

$$y - 2x = cx^2y.$$

## PART – IV

41. a.

$$T = \begin{array}{c c} & \begin{array}{cc} P & Q \end{array} \\ \begin{array}{c} P \\ Q \end{array} & \begin{bmatrix} 0.8 & 0.2 \\ 0.6 & 0.4 \end{bmatrix} \end{array}$$

Shares after one week

$$\begin{array}{cc} P & Q \\ (0.7 & 0.3) \end{array} \begin{array}{c c} & \begin{array}{cc} P & Q \end{array} \\ \begin{array}{c} P \\ Q \end{array} & \begin{bmatrix} 0.8 & 0.2 \\ 0.6 & 0.4 \end{bmatrix} \end{array} = \begin{array}{cc} P & Q \\ (0.74 & 0.26) \end{array}$$

$P = 74\%$ $Q = 26\%$

Shares after two weeks

$$\begin{array}{cc} P & Q \\ (0.74 & 0.26) \end{array} \begin{array}{c c} & \begin{array}{cc} P & Q \end{array} \\ \begin{array}{c} P \\ Q \end{array} & \begin{bmatrix} 0.8 & 0.2 \\ 0.6 & 0.4 \end{bmatrix} \end{array} = \begin{array}{cc} P & Q \\ (0.748 & 0.252) \end{array}$$

$P = 74.8\%$, $Q = 25.2\%$

At Equilibrium,

$(P\ Q)\,T = (P\ Q)$ where $P + Q = 1$

$$(P\ Q)\begin{bmatrix} 0.8 & 0.2 \\ 0.6 & 0.4 \end{bmatrix} = (P\ Q)$$

$0.8P + 0.6Q = P \Rightarrow P = 0.75$

Equilibrium is reached when P's share is 75% and Q's share is 25%.

b. $P = \frac{10}{100000} = 0.0001$

$n = 4000$

$\lambda = np = 0.4$

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-0.4}(0.4)^x}{x!}$$

$$P\left(\begin{array}{c}\text{more than} \\ \text{3 injured}\end{array}\right) = P(x > 3)$$

$$= 1 - P(X \le 3)$$

$$= 1 - e^{-0.4}\left[\frac{1}{1} + \frac{0.4}{1} + \frac{0.16}{2} + \frac{0.064}{6}\right]$$

$$= 0.0006$$

42. a. Control limits for $\bar{X}$ chart

$\bar{\bar{X}} = \frac{1}{n}\Sigma\bar{X} = 10.66$

$\bar{R} = \frac{1}{n}\Sigma R = 6.3$

$U.C.L = \bar{\bar{X}} + A_2\bar{R} = 14.295$

$L.C.L = \bar{\bar{X}} - A_2\bar{R} = 7.025$

$C.L = \bar{\bar{X}} = 10.66$

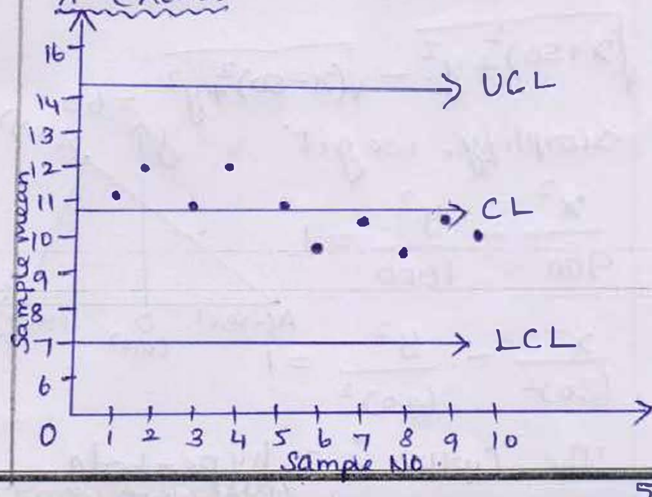
Range Chart

$U.C.L = D_4\bar{R} = 13.324$

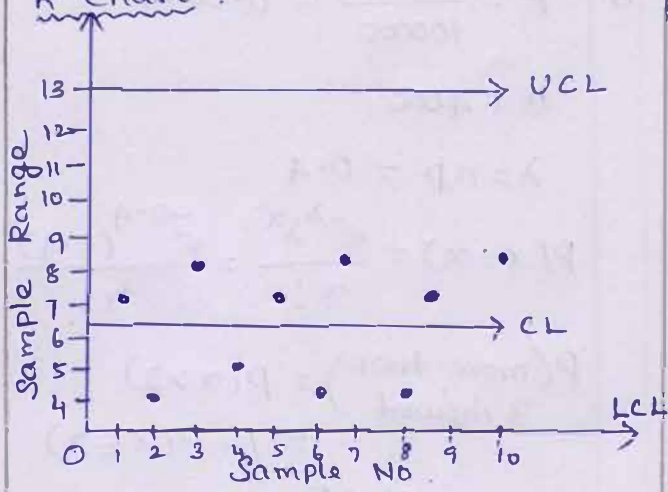
$L.C.L = D_3\bar{R} = 0$

$C.L = \bar{R} = 6.3$

$\bar{X}$ Chart



R Chart :




Conclusion :-

Since all the points of sample mean and range are within the control limits, the process is in control.

b. Let the cost per unit at $B = C$

the cost per unit at $A = C - 12$

Delivery cost per unit from A to P $= \frac{20}{100} AP$

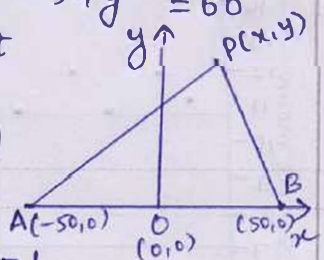
Delivery cost per unit from B to P $= \frac{20}{100} BP$

$(C-12) + \frac{20}{100}(AP) = C + \frac{20}{100}(BP)$

$AP - BP = 60$

$\sqrt{(x+50)^2 + y^2} - \sqrt{(x-50)^2 + y^2} = 60$

Simplify, we get




$\frac{x^2}{900} - \frac{y^2}{1600} = 1$

$\frac{x^2}{(30)^2} - \frac{y^2}{(40)^2} = 1$

The curve is a hy

43 a. $x^2 + 4y^2 - 8x - 16y - 68 = 0$ — ①

$\frac{dy}{dx} = \frac{4-x}{4y-8}$

i) Tangent is $\perp^r$ to x-axis

$\frac{dx}{dy} = 0 \Rightarrow 4y - 8 = 0 \Rightarrow y = 2$

when $y = 2 \Rightarrow x = 14, -6$

$\therefore$ The point is $(14, 2), (-6, 2)$

ii) Tangent is $\perp^r$ to y-axis

$\frac{dy}{dx} = 0 \Rightarrow x = 4$

when $x = 4 \Rightarrow y = -3, 7$

$\therefore$ The point is

$(4, 7)\ (4, -3)$.

b. Let $I = \int_0^{\pi/4} \log(1 + \tan x)\, dx$

$= \int_0^{\pi/4} \log(1 + \tan(\pi/4 - x))\, dx$

$= \int_0^{\pi/4} \log\left(1 + \frac{1 - \tan x}{1 + \tan x}\right) dx$

$= \int_0^{\pi/4} \log\left(\frac{2}{1 + \tan x}\right) dx$

$I = \int_0^{\pi/4} \log 2\, dx - \int_0^{\pi/4} \log(1 + \tan x)\, dx$

$2I = \log 2\, [x]_0^{\pi/4}$

$I = \frac{\pi}{8} \log 2$

44 a.

$$B = \begin{bmatrix} \frac{1}{8} & \frac{1}{5} \\ \frac{1}{4} & \frac{1}{20} \end{bmatrix}$$

$$I-B = \begin{bmatrix} 7/8 & -1/5 \\ -1/4 & \frac{19}{20} \end{bmatrix}$$ the main diagonal elements are +ve.

$$|I-B| = \frac{125}{160} > 0$$

$$(I-B)^{-1} = \frac{160}{125}\begin{bmatrix} \frac{19}{20} & \frac{1}{5} \\ \frac{1}{4} & \frac{7}{8} \end{bmatrix}$$

$$X = (I-B)^{-1} D$$

$$= \begin{pmatrix} 7104 \\ 6080 \end{pmatrix}$$

The output for P is 7104 and for Q is 6080.

b. The auxiliary equation is

$$m^2 + 10m + 25 = 0$$

$$m = -5, -5$$

$$C.F = (Ax+B)\, e^{-5x}$$

$$P.I_1 = \frac{1}{10}, \quad P.I_2 = \frac{x^2}{2} e^{-5x}$$

The solution is

$$y = (Ax+B)e^{-5x} + \frac{1}{10} + \frac{x^2}{2}e^{-5x}$$

45 a. Let $f(x,y) = \sin u = \dfrac{\sqrt{x} - \sqrt{y}}{\sqrt[3]{x} + \sqrt[3]{y}}$

$$f(tx, ty) = t^{1/6} f(x,y)$$

$f$ is a homogeneous function of degree $1/6$.

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{6}\tan u$$

b. $x = 2005$, $x_n = 2011$, $h = 10$

$$P = \frac{x - x_n}{h} = -0.6$$

| $x$ | $y_n$ | $\nabla y_n$ | $\nabla^2 y_n$ | $\nabla^3 y_n$ | $\nabla^4 y_n$ |
|---|---|---|---|---|---|
| 1971 | 56 | | | | |
| | | 20 | | | |
| 1981 | 76 | | -5 | | |
| | | 15 | | 2 | |
| 1991 | 91 | | -3 | | -3 |
| | | 12 | | -1 | |
| 2001 | 103 | | -4 | | |
| | | 8 | | | |
| 2011 | 111 | | | | |

$$y = y_n + \frac{P}{1!}\nabla y_n + \frac{P(P+1)}{2!}\nabla^2 y_n + \frac{P(P+1)(P+2)}{3!}\nabla^3 y_n + \cdots$$

$$= 106.8368$$

∴ The population year 2005 is 106.8368

46 a. $\dfrac{dc}{dq} = \dfrac{c^2 + 2cq}{q^2}$

$$c = vq$$

$$\frac{dc}{dq} = v + q\frac{dv}{dq}$$

$$\frac{dv}{v(v+1)} = \frac{dq}{q}$$

$$\int \frac{(v+1)-v}{v(v+1)}\,dv = \int \frac{dq}{q} + k$$

$$\int \frac{dv}{v} - \int \frac{dv}{v+1} = \int \frac{dq}{q} + \log k$$

$$\log v - \log(v+1) = \log q + \log k$$

$$c = kq(c+q)$$

when $c = 1$ and $q = 1 \Rightarrow k = 1/2$

$c = \dfrac{q(c+q)}{2}$, relation between $c$ and $q$

b. $q_1 = 240 - P_1^2 + 6P_2 - P_1P_2$

$$\frac{\partial q_1}{\partial P_1} = -2P_1 - P_2, \quad \frac{\partial q_1}{\partial P_2} = 6 - P_1$$

$$\frac{Eq_1}{EP_1} = \frac{-P_1}{q_1}\frac{\partial q_1}{\partial P_1}$$

$$= \frac{-P_1}{240 - P_1^2 + 6P_2 - P_1P_2}(-2P_1 - P_2)$$

when $P_1 = 5$, $P_2 = 4$, $\frac{Eq_1}{EP_1} = \frac{70}{219}$

$$\frac{Eq_1}{EP_2} = \frac{-P_2}{q_1}\frac{\partial q_1}{\partial P_2} = \frac{-P_2(6-P_1)}{240 - P_1^2 + 6P_2 - P_1P_2}$$

when $P_1 = 5$ and $P_2 = 4$

$$\frac{Eq_1}{EP_2} = \frac{-4}{219}$$

47.
a. $n = 1600$  $\bar{x} = 99$

$\mu = 100$, $\sigma = 15$

$H_0 : \mu = 100$

$H_1 : \mu \neq 100$

Test statistic, $z$ is the standard normal variate under $H_0$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = -2.67$$

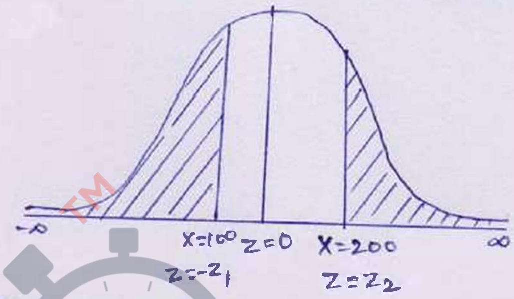
$|z| = 2.67$

since $|z| = 2.67 > 1.96$

Acceptance region is $|z| < 1.96$, $H_0$ is rejected at 5% level of significance.

$\therefore$ The sample is not from this population.

b.




From the diagram,

$P(-z_1 < z < 0) = 0.3$

$z_1 = 0.84$

$$-0.84 = \frac{100-\mu}{\sigma}$$

$100 - \mu = -0.84\sigma$ — (1)

$P(0 < z < z_2) = 0.2$

$z_2 = 0.525$

$$0.525 = \frac{200-\mu}{\sigma}$$

$200 - \mu = 0.525\sigma$ — (2)

Solve (1) and (2)

$\mu = 161.53$, $\sigma = 73.26$