HIGHER SECONDARY SECOND YEAR
MATHEMATICS
MODEL QUESTION PAPER 2019 - 20

Time Allowed: 15 Minutes + 2.30 Hours] **[Maximum Marks:90**

Instructions: (a) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(b) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

PART – I

Note: (i) All questions are **compulsory.** $20\times1=20$

(ii) Choose the most suitable answer from the given **four** correct alternatives and write the option code with the corresponding answer.

1. If $A=\begin{bmatrix}3 & 5\\1 & 2\end{bmatrix}$, $B=adj A$ and $C=3A$, then $\frac{|adj B|}{|C|}=$

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{4}$ (d) 1

2. If the inverse of the matrix $\begin{bmatrix}1 & 2\\3 & -5\end{bmatrix}$ is $\frac{1}{11}\begin{bmatrix}a & b\\c & d\end{bmatrix}$, then the ascending order of a,b,c,d is

(a) a,b,c,d (b) d,b,c,a (c) c,a,b,d (d) b,a,c,d

3. The least value of n satisfying $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^n=1$ is

(a) 30 (b) 24 (c) 12 (d) 18

4. The principal argument of $\frac{3}{-1+i}$ is

(a) $\frac{-5\pi}{6}$ (b) $\frac{-2\pi}{3}$ (c) $\frac{-3\pi}{4}$ (d) $\frac{-\pi}{2}$

5. The polynomial equation $x^3+2x+3=0$ has

(a) one negative and two real roots (b) one positive and two imaginary roots

(c) three real roots (d) no solution

6. The domain of the function defined by $f(x)=\sin^{-1}\left(\sqrt{x-1}\right)$ is

(a) $[1,2]$ (b) $[-1,1]$ (c) $[0,1]$ (d) $[-1,0]$

7. If $x+y=k$ is a normal to the parabola $y^2=12x$, then the value of k is

(a) 3 (b) -1 (c) 1 (d) 9

8. The circle passing through $(1,-2)$ and touching the x-axis at $(3,0)$, again passing through the point is

(a) $(-5,2)$ (b) $(2,-5)$ (c) $(5,-2)$ (d) $(-2,5)$

9. The volume of the parallelepiped with its edges represented by the vectors $\hat{i}+\hat{j}, \hat{i}+2\hat{j}, \hat{i}+\hat{j}+\pi\hat{k}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{\pi}{4}$

10. If the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lies in the plane $x+3y-\alpha z+\beta=0$, then (α,β) is

(a) $(-5,5)$ (b) $(-6,7)$ (c) $(5,-5)$ (d) $(6,-7)$

11. The function $\sin^4 x+\cos^4 x$ is increasing in the interval

(a) $\left[\frac{5\pi}{8},\frac{3\pi}{4}\right]$ (b) $\left[\frac{\pi}{2},\frac{5\pi}{8}\right]$ (c) $\left[\frac{\pi}{4},\frac{\pi}{2}\right]$ (d) $\left[0,\frac{\pi}{4}\right]$

12. The curve $y=ax^4+bx^2$ with $ab>0$

(a) has no horizontal tangent (b) is concave up

(c) is concave down (d) has no points of inflection

13. If $u=(x-y)^2$, then $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}$ is

(a) 1 (b) -1 (c) 0 (d) 2

14. The value of $\int_0^{\pi}\frac{dx}{1+5^{\cos x}}$ is

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{2}$ (d) 2π

15. The volume of solid of revolution of the region bounded by $y^2=x(a-x)$ about x-axis is

(a) πa^3 (b) $\frac{\pi a^3}{4}$ (c) $\frac{\pi a^3}{5}$ (d) $\frac{\pi a^3}{6}$

16. If m,n are the order and degree of the differential equation $\left(\frac{d^4y}{dx^4}+\frac{d^2y}{dx^2}\right)^{\frac{1}{2}}=a\frac{d^3y}{dx^3}$ respectively, then the value of $4m-n$ is

(a) 15 (b) 12 (c) 14 (d) 13

17. The solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}+\frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ is

(a) $x\phi\left(\frac{y}{x}\right)=k$ (b) $\phi\left(\frac{y}{x}\right)=kx$ (c) $y\phi\left(\frac{y}{x}\right)=k$ (d) $\phi\left(\frac{y}{x}\right)=ky$

18. A random variable X has the following distribution.

x	1	2	3	4
$P(X=x)$	c	$2c$	$3c$	$4c$

Then the value of c is

(a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4

19. If $P\{X=0\}=1-P\{X=1\}$ and $E[X]=3Var(X)$, then $P\{X=0\}$ is

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) $\frac{1}{3}$

20. Which one is the contrapositive of the statement $(p\vee q)\to r$

(a) $\neg r\to(\neg p\wedge\neg q)$ (b) $\neg r\to(p\vee q)$

(c) $r\to(p\wedge q)$ (d) $p\to(q\vee r)$

PART – II

Note: (i) Answer any **SEVEN** questions. $7\times2=14$

(ii) **Question number 30 is compulsory.**

21. Solve the following system of linear equations by Cramer's rule : $2x-y=3, x+2y=-1$.

22. If z_1, z_2 and z_3 are complex numbers such that $|z_1|=|z_2|=|z_3|=|z_1+z_2+z_3|=1$, find the value of $\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|$.

23. Find the value of $\sin\left(\frac{\pi}{3}+\cos^{-1}\left(-\frac{1}{2}\right)\right)$.

24. Find the equation of the parabola with vertex $(-1,-2)$, axis parallel to y-axis and passing through $(3,6)$.

25. If $\vec{a},\vec{b},\vec{c}$ are three unit vectors such that $\vec{b}$ and $\vec{c}$ are non-parallel and $\vec{a}\times(\vec{b}\times\vec{c})=\frac{1}{2}\vec{b}$, find the angle between $\vec{a}$ and $\vec{c}$.

26. If the mass $m(x)$ (in kilogram) of a thin rod of length x (in meters) is given by, $m(x)=\sqrt{3x}$ then what is the rate of change of mass with respect to the length when it is $x=27$ meters?

27. Evaluate : $\int_0^\infty e^{-ax}x^n dx$, where $a>0$.

28. Show that $y=ax+\frac{b}{x}, x\neq0$ is a solution of the differential equation $x^2y''+xy'-y=0$.

29. Find the mean of a random variable X, whose probability density function is

$$f(x)=\begin{cases}\lambda e^{-\lambda x} & \text{for } x\geq0\\ 0 & \text{otherwise}\end{cases}$$

30. Let $*$ be a binary operation on set Q of rational numbers defined as $a*b=\frac{ab}{8}$. Write the identity for $*$, if any.

PART – III

Note: (i) Answer any **SEVEN** questions. $7\times3=21$

(ii) **Question number 40 is compulsory.**

31. Find the inverse of $\begin{bmatrix}2 & -1\\ 5 & -2\end{bmatrix}$ by Gauss Jordan method.

32. If $\omega\neq1$ is a cube root of unity, show that the roots of the equation $(z-1)^3+8=0$ are $-1, 1-2\omega, 1-2\omega^2$.

33. Find all real numbers satisfying $4^x - 3(2^{x+2}) + 2^5 = 0$.

34. Find the centre, foci, and eccentricity of the hyperbola $12x^2 - 4y^2 - 24x + 32y - 127 = 0$.

35. Find the image of the point whose position vector is $\hat{i} + 2\hat{j} + 3\hat{k}$ in the plane $\vec{r} \cdot (\hat{i} + 2\hat{j} + 4\hat{k}) = 38$.

36. Evaluate : $\lim_{x \to 0^+} x \log x$.

37. Find a linear approximation for the function given below at the indicated points.
$f(x) = x^3 - 5x + 12,\ x_0 = 2$.

38. By using the properties of definite integrals, evaluate $\int_0^3 |x-1|\,dx$

39. Solve : $\frac{dy}{dx} + 2y \cot x = 3x^2 \operatorname{cosec}^2 x$.

40. A fair coin is tossed a fixed number of times.If the probability of getting seven heads is equal to that of getting nine heads, find the probability of getting exactly two heads.

PART – IV

Note: Answer all the questions. $7 \times 5 = 35$

41. (a) By using Gaussian elimination method, balance the chemical reaction equation:
$C_2H_6 + O_2 \to H_2O + CO_2$.

(OR)

(b) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$

42. (a) Solve the equation : $3x^4 - 16x^3 + 26x^2 - 16x + 3 = 0$.

(OR)

(b) Solve : $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$.

43. (a) A rod of length $1.2m$ moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is $0.3m$ from the end in contact with x-axis is an ellipse. Find the eccentricity.

(OR)

(b) Find the non-parametric and Cartesian equations of the plane passing through the point (4, 2, 4) , and is perpendicular to the planes $2x + 5y + 4z + 1 = 0$ and $4x + 7y + 6z + 2 = 0$.

44. (a) A steel plant is capable of producing x tonnes per day of a low-grade steel and y tonnes per day of a high-grade steel, where $y = \frac{40-5x}{10-x}$. If the fixed market price of low-grade steel is half that of high-grade steel, then what should be optimal productions in low-grade steel and high-grade steel in order to have maximum receipts.

(OR)

(b) Let $z(x,y) = xe^{y} + ye^{-x}, x = e^{-t}, y = st^{2}, s,t \in \mathbb{R}$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

45. (a) Find the area of the region bounded between the parabola $x^2 = y$ and the curve $y = |x|$.

(OR)

(b) Water at temperature $100^\circ C$ cools in 10 minutes to $80^\circ C$ in a room temperature of $25^\circ C$. Find

(i) The temperature of water after 20 minutes

(ii) The time when the temperature is $40^\circ C$ $\left[\log_e \frac{11}{15} = -0.3101;\ \log_e 5 = 1.6094\right]$

46. (a) Suppose a discrete random variable can take only the values 0, 1, and 2. The probability mass function is defined by

$$f(x) = \begin{cases} \frac{x^2+1}{k}, & \text{for } x = 0,1,2 \\ 0 & \text{otherwise} \end{cases}$$

Find (i) the value of k (ii) cumulative distribution function (iii) $P(x \geq 1)$

(OR)

(b) Using truth table check whether the statements $\neg(p \vee q) \vee (\neg p \wedge q)$ and $\neg p$ are logically equivalent.

47. (a) Prove by vector method that $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$.

(OR)

(b) Find the equations of tangent and normal to the curve $y^2 - 4x - 2y + 5 = 0$ at the point where it cuts the x- axis.