ALGEBRA

Points to Remember

- A system of linear equations in three variables will be according to one of the following cases.
  (i) Unique solution (ii) Infinitely many solutions (iii) No solution
- The least common multiple of two or more algebraic expressions is the expression of lowest degree (or power) such that the expressions exactly divides it.
- A polynomial of degree two in variable x is called a quadratic polynomial in x. Every quadratic polynomial can have atmost two zeroes. Also the zeroes of a quadratic polynomial intersects the x-axis.
- The roots of the quadratic equation $ax^2 + bx + c = 0$, $(a \neq 0)$ are given by $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
- For a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$

  Sum of the roots $\alpha + \beta = \dfrac{-b}{a} = \dfrac{-\text{Co-efficient of } x}{\text{Co-efficient of } x^2}$

  Product of the roots $\alpha\beta = \dfrac{c}{a} = \dfrac{\text{Constant term}}{\text{Co-efficient of } x^2}$
- If the roots of a quadratic equation are α and β, then the equation is given by $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.
- The value of the discriminant $(\Delta = b^2 - 4ac)$ decides the nature of roots as follows
  (i) When $\Delta > 0$, the roots are real and unequal.
  (ii) When $\Delta = 0$, the roots are real and equal.
  (iii) When $\Delta < 0$, there are no real roots.
- Solving quadratic equation graphically.
- A matrix is a rectangular array of elements arranged in rows and columns.
- Order of a matrix

  If a matrix A has m number of rows and n number of columns, then the order of the matrix A is (Number of rows) $\times$ (Number of columns) that is, $m \times n$. We read $m \times n$ as m cross n or m by n. It may be noted that $m \times n$ is not a product of m and n.
- Types of matrices
  (i) A matrix is said to be a row matrix if it has only one row and any number of columns. A row matrix is also called as a row vector.
  (ii) A matrix is said to be a column matrix if it has only one column and any number of rows. It is also called as a column vector.

(iii) A matrix in which the number of rows is equal to the number of columns is called a square matrix.

(iv) A square matrix, all of whose elements, except those in the leading diagonal are zero is called a diagonal matrix.

(v) A diagonal matrix in which all the leading diagonal elements are same is called a scalar matrix.

(vi) A square matrix in which elements in the leading diagonal are all "1" and rest are all zero is called an identity matrix (or) unit matrix.

(vii) A matrix is said to be a zero matrix or null matrix if all its elements are zero.

(viii) If A is a matrix, the matrix obtained by interchanging the rows and columns of A is called its transpose and is denoted by A^T.

(ix) A square matrix in which all the entries above the leading diagonal are zero is called a lower triangular matrix.

If all the entries below the leading diagonal are zero, then it is called an upper triangular matrix.

(x) Two matrices A and B are said to be equal if and only if they have the same order and each element of matrix A is equal to the corresponding element of matrix B. That is, $a_{ij} = b_{ij}$ for all i, j.

- The negative of a matrix $A_{m\times n}$ denoted by $-A_{m\times n}$ is the matrix formed by replacing each element in the matrix $A_{m\times n}$ with its additive inverse.

- **Addition and subtraction of matrices**

  Two matrices can be added or subtracted if they have the same order. To add or subtract two matrices, simply add or subtract the corresponding elements.

- **Multiplication of matrix by a scalar**

  We can multiply the elements of the given matrix A by a non-zero number k to obtain a new matrix kA whose elements are multiplied by k. The matrix kA is called scalar multiplication of A.

  Thus if $A = (a_{ij})_{m\times n}$ then , $kA = (ka_{ij})_{m\times n}$ for all $i = 1,2,\ldots,m$ and for all $j = 1,2,\ldots,n$.