NUMBERS AND SEQUENCES

Points to Remember

- **Euclid's division lemma**

  If a and b are two positive integers then there exist unique integers q and r such that $a = bq + r,\ 0 \le r < |b|$

- **Fundamental theorem of arithmetic**

  Every composite number can be expressed as a product of primes and this factorization is unique except for the order in which the prime factors occur.

- **Arithmetic Progression**

  (i) Arithmetic Progression is $a,\ a+d,\ a+2d,\ a+3d, \ldots$ n^{th} term is given by $t_n = a + (n-1)d$

  (ii) Sum to first n terms of an A.P. is $S_n = \frac{n}{2}[2a + (n-1)d]$

  (iii) If the last term l (n^{th} term is given, then $S_n = \frac{n}{2}[a + 1]$)

- **Geometric Progression**

  (i) Geometric Progression is $a,\ ar,\ ar^2, \ldots, ar^{n-1}$. n^{th} term is given by $t_n = ar^{n-1}$

  (ii) Sum to first n terms of an G.P. is $S_n = \frac{a(r^n - 1)}{r-1}$ if $r \ne 1$

  (iii) Suppose $r = 1$ then $S_n = na$

  (iv) Sum to infinite terms of a G.P. $a + ar + ar^2 + \cdots$ is $S = \frac{a}{1-r}$, where $-1 < r < 1$

- **Special Series**

  (i) The sum of first n natural numbers $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$

  (ii) The sum of squares of first n natural numbers

  $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

  (iii) The sum of cubes of first n natural numbers $1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

  (iv) The sum of first n odd natural numbers $1 + 3 + 5 + \cdots + (2n-1) = n^2$